# Lagrangian Relaxation

## Extensive form

The extensive form of the standard problem for four investment periods is:

Minimize:

Subject to:



## Extensive form with duplicate variables and complicating equality constraints

Minimize:

Subject to:



The additional blocks/constraints in the lower left corner enforce equality constraints between the duplicate investment variables. These equality constraints are dualized in the lagrangian relaxation. Notice that only one copy of each duplicated investment variable appears in the objective function.

## Extensive form with duplicate variables and dualized equality constraints

Minimize:

The second and third line reflect the dualized equality constraints for duplicated first and second period investment decision variables. The second line is broken into three sections to represent how each copy of the first-period investment decisions can deviate from the average. The equality constraints could also be represented in a pair-wise manner, such as (for period 1 decisions):

I chose not to do this because it increases the number of constraints and variables that have to be tracked and passed into each subproblem.

Subject to:



The objective function can be rearranged so each sub-problem appears on a different line:

Minimize:

### Ideas/Questions:

* The general standard method of Lagrangian relaxation uses a separate penalty term for each dualized constraint. I don’t know if it is possible to use a single term for all of the first stage investment deviations.
* I might be able to bootstrap by sequentially solving the subproblems..
* Start with first period’s sub-problem with deviation penalties of 0 and use those values of as an estimate of
* Solve the second period sub-problem with p2 deviation penalties of 0, and pass the solution’s value of as an estimate of . But what values should I use for p1 deviation penalties?
* Repeat for stage 3
* Combine results of all sub-problems to calculate and , start the normal Lagrangian iterative process.
* Another bootstrapping method could be using the capital costs for initial values of penalty terms, and use starting values of 0 for the average investment costs and

## Decomposed form with duplicate variables and dualized equality constraints

### Period 1

Minimize:

Subject to:



### Period 2

Minimize:

Subject to:



### Period 3

Minimize:

Subject to:



### Period 4

Minimize:

Subject to:



# Implementation notes

I have all these clauses in the Lagrangian objective function and problem statement that limit costs and considered hours to the target period. I don’t like them because it makes writing and maintaining code more time consuming and leaves lots of places for small typo bugs to creep in.

The restrictions on investment decisions are different in the problem statement and objective function. In the problem statement, I restrict investment decisions to all periods up to and including the target period: p <= target\_period. This ensures I have a copy of investment decisions from each prior period. However, these copies should not appear in the objective function to avoid double-counting their costs. So in the objective function, I restrict the investment decisions that count to the target period: p = target\_period

The restrictions on dispatch decisions are the same in the problem statement and objective function because I don’t need copies of the dispatch decisions from other investment periods. I don’t Have to remove other period dispatch decisions from the objective function; if I leave them in place, they will count towards fixed costs, but won’t be decision variables and won’t complicate the problem cplex is solving. The most significant downside of leaving them in place is the objective function appears irregular since dispatch and investments have different tweaks.