# Lagrangian Relaxation

## Extensive form

The extensive form of the standard problem for three investment periods is:

Minimize:

Subject to:



## Extensive form with duplicate variables and complicating equality constraints

Minimize:

Subject to:



The additional blocks/constraints in the lower left corner enforce equality constraints between the duplicate investment variables. These equality constraints are dualized in the lagrangian relaxation. Notice that only one copy of each duplicated investment variable appears in the objective function.

## Extensive form with duplicate variables and dualized equality constraints

Minimize:

The second and third line reflect the dualized equality constraints for duplicated first and second period investment decision variables. The second line is broken into three sections to represent how each copy of the first-period investment decisions can deviate from the average. The equality constraints could also be represented in a pair-wise manner, such as:

I chose not to do this because it increases the data that has to be passed into each subproblem.

Subject to:



The objective function can be rearranged so each sub-problem appears on a different line:

Minimize:

### Ideas/Questions:

* The general standard method of Lagrangian relaxation uses a separate penalty term for each dualized constraint. I don’t know if it is possible to use a single term for all of the first stage investment deviations.
* I might be able to bootstrap by sequentially solving the subproblems..
* Start with first period’s sub-problem with deviation penalties of 0 and use those values of as an estimate of
* Solve the second period sub-problem with p2 deviation penalties of 0, and pass the solution’s value of as an estimate of . But what values should I use for p1 deviation penalties?
* Repeat for stage 3
* Combine results of all sub-problems to calculate and , start the normal Lagrangian iterative process.
* Another bootstrapping method could be using the capital costs for initial values of penalty terms, and use starting values of 0 for the average investment costs and

## Decomposed form with duplicate variables and dualized equality constraints

### Period 1

Minimize:

Subject to:



### Period 2

Minimize:

Subject to:



### Period 3

Minimize:

Subject to:

